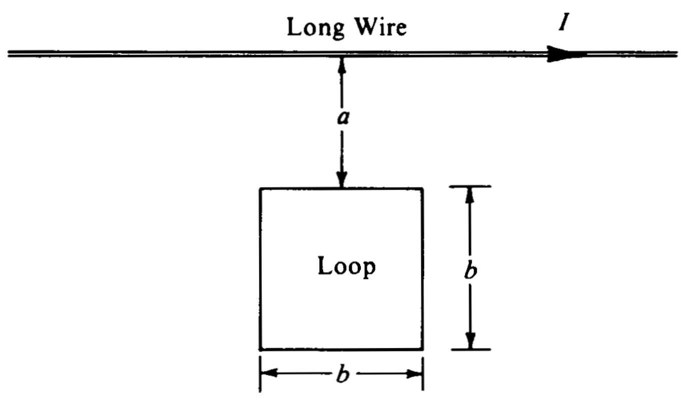
PHYS 2212
Recitation 10 April 5–8

Example
A long wire carries a current in the direction shown. The current I varies linearly with time t as follows:

$$I = ct$$

where c is a positive constant. The long wire is in the same plane as a square loop of wire of side b, as shown in the diagram. The side of the loop nearest the long wire is parallel to it and a distance a from it. The loop has a resistance R and is fixed in space.

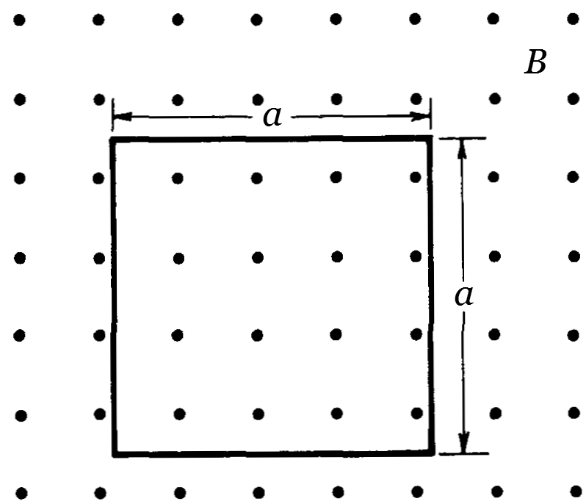
Determine the induced current in the loop as a function of time and indicate the direction of the induced current in the loop on the diagram.



PHYS 2212 Section: ______ Name: ______________________
Recitation 10 April 5–8

Problem

A square wire loop of resistance R and side of length a lies in the plane of the page, as shown. The loop is in a magnetic field $\vec{B}$ that is directed out of the page. At time $t = 0$, the field has a magnitude B_0; it then decreases according to the equation $B = B_0 e^{-\alpha t}$, where α is a positive constant.



a) Find an expression for the current induced in the loop for time $t > 0$. Indicate its direction.

b) Find an expression for the total energy dissipated as heat during the time from zero to infinity.

PHYS 2212 **Solution**
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Example
A long wire carries a current in the direction shown. The current I varies linearly with time t as follows:

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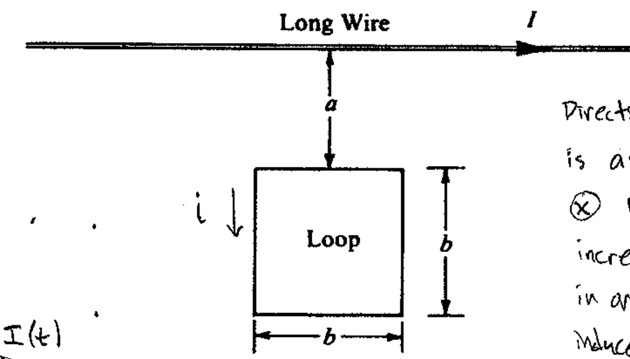
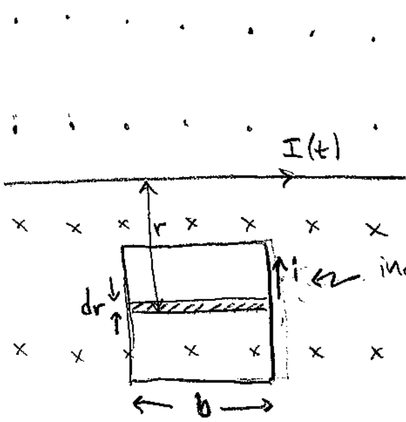


Direction of induced current is as shown because the inward ⊗ magnetic field in the loop is increasing. Lenz's law tells us that in order to oppose this increase the induced current creates a magnetic field that opposes the change.



induced current is caused by induced emf: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cdot b\,dr = \int_{r=a}^{r=a+b} \frac{\mu_0 I}{2\pi r}\, b\,dr$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_a^{a+b} \frac{1}{r}\,dr$$

At distance r from long wire:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

but $\Phi_B = \Phi_B(t)$ because $I = I(t) = ct$

$$\Phi_B = \frac{\mu_0 b c}{2\pi} \ln\left(\frac{a+b}{a}\right) \cdot t$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 b c}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

Induced current: $\boxed{i = \frac{\mathcal{E}}{R} = \frac{\mu_0 b c}{2\pi R} \ln\left(\frac{a+b}{a}\right)}$ and it flows counterclockwise through the square loop

PHYS 2212 **Solution**

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Problem

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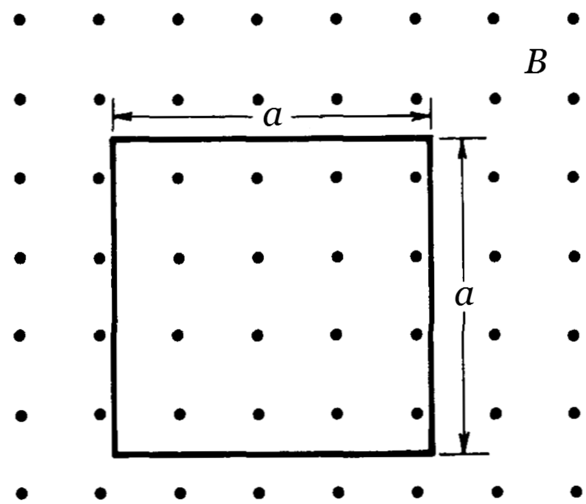

a) Find an expression for the current induced in the loop for time $t > 0$. Indicate its direction.

b) Find an expression for the total energy dissipated as heat during the time from zero to infinity.

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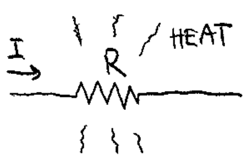
a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ $\qquad \Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA = B \int dA = BA = (B_0 e^{-\alpha t})(a^2)$

$$\mathcal{E} = -\frac{d}{dt}\left(B_0 a^2 e^{-\alpha t}\right) = -(B_0 a^2)(-\alpha)\,e^{-\alpha t} = B_0 a^2 \alpha\, e^{-\alpha t}$$

$$\boxed{I = \frac{\mathcal{E}}{R} = \frac{B_0 a^2 \alpha\, e^{-\alpha t}}{R}}$$

← this induced current flows counterclockwise by Lenz's law ("nature abhors a change in flux")

b)



Rate of heating = power $\quad P = I^2 R$, $\quad I = I(t)$

$P = \dfrac{dE}{dt}$

$$E = \int P\,dt$$

$$E_{tot} = \int_0^\infty I^2 R\,dt = \int_0^\infty \left[\frac{B_0 a^2 \alpha e^{-\alpha t}}{R}\right]^2 R\,dt = \frac{B_0^2 a^4 \alpha^2}{R} \int_0^\infty e^{-2\alpha t}\,dt$$

$$= \frac{B_0^2 a^4 \alpha^2}{R} \left[\frac{-1}{2\alpha} e^{-2\alpha t}\right]_0^\infty = -\frac{B_0^2 a^4 \alpha^2}{2\alpha R}\left[e^{-\infty} - e^0\right]$$

$$\boxed{E_{tot} = \frac{B_0^2 a^4 \alpha}{2R}}$$